Cambridge Assessment International Education
Cambridge International Advanced Level

FURTHER MATHEMATICS **9231/11**

Paper 1 **May/June 2018**

MARK SCHEME

Maximum Mark: 100

Published

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

Cambridge International will not enter into discussions about these mark schemes.

Cambridge International is publishing the mark schemes for the May/June 2018 series for most Cambridge IGCSE™, Cambridge International A and AS Level and Cambridge Pre-U components, and some Cambridge O Level components.

This document consists of **21** printed pages.

Generic Marking Principles

These general marking principles must be applied by all examiners when marking candidate answers. They should be applied alongside the specific content of the mark scheme or generic level descriptors for a question. Each question paper and mark scheme will also comply with these marking principles.

GENERIC MARKING PRINCIPLE 1:

Marks must be awarded in line with:

- the specific content of the mark scheme or the generic level descriptors for the question
- the specific skills defined in the mark scheme or in the generic level descriptors for the question
- the standard of response required by a candidate as exemplified by the standardisation scripts.

GENERIC MARKING PRINCIPLE 2:

Marks awarded are always **whole marks** (not half marks, or other fractions).

GENERIC MARKING PRINCIPLE 3:

Marks must be awarded **positively**:

- marks are awarded for correct/valid answers, as defined in the mark scheme. However, credit is given for valid answers which go beyond the scope of the syllabus and mark scheme, referring to your Team Leader as appropriate
- marks are awarded when candidates clearly demonstrate what they know and can do
- marks are not deducted for errors
- marks are not deducted for omissions
- answers should only be judged on the quality of spelling, punctuation and grammar when these features are specifically assessed by the question as indicated by the mark scheme. The meaning, however, should be unambiguous.

GENERIC MARKING PRINCIPLE 4:

Rules must be applied consistently e.g. in situations where candidates have not followed instructions or in the application of generic level descriptors.

GENERIC MARKING PRINCIPLE 5: Marks should be awarded using the full range of marks defined in the mark scheme for the question (however; the use of the full mark range may be limited according to the quality of the candidate responses seen).
GENERIC MARKING PRINCIPLE 6: Marks awarded are based solely on the requirements as defined in the mark scheme. Marks should not be awarded with grade thresholds or grade descriptors in mind.

Mark Scheme Notes

Marks are of the following three types:

M Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.

A Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).

B Mark for a correct result or statement independent of method marks.

- When a part of a question has two or more 'method' steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly when there are several B marks allocated. The notation DM or DB (or dep*) is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.

- The symbol FT implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only. A and B marks are not given for fortuitously "correct" answers or results obtained from incorrect working.

  - Note: B2 or A2 means that the candidate can earn 2 or 0.
    B2/1/0 means that the candidate can earn anything from 0 to 2.

  The marks indicated in the scheme may not be subdivided. If there is genuine doubt whether a candidate has earned a mark, allow the candidate the benefit of the doubt. Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored.

- Wrong or missing units in an answer should not lead to the loss of a mark unless the scheme specifically indicates otherwise.

- For a numerical answer, allow the A or B mark if a value is obtained which is correct to 3 s.f., or which would be correct to 3 s.f. if rounded (1 d.p. in the case of an angle). As stated above, an A or B mark is not given if a correct numerical answer arises fortuitously from incorrect working. For Mechanics questions, allow A or B marks for correct answers which arise from taking g equal to 9.8 or 9.81 instead of 10.

The following abbreviations may be used in a mark scheme or used on the scripts:

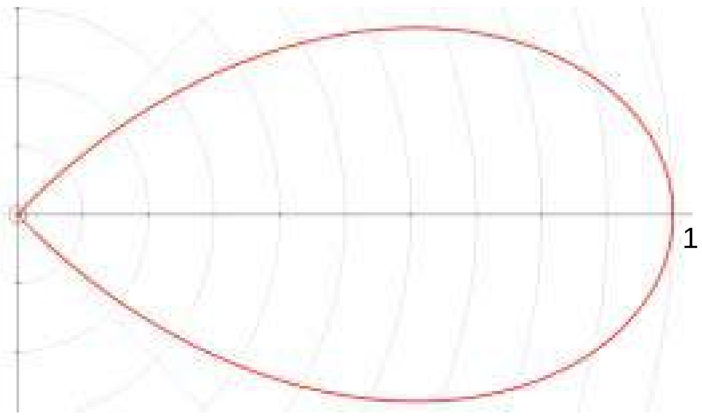
AEF/OE	Any Equivalent Form (of answer is equally acceptable) / Or Equivalent
AG	Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)
CAO	Correct Answer Only (emphasising that no ‘follow through’ from a previous error is allowed)
CWO	Correct Working Only – often written by a ‘fortuitous’ answer
ISW	Ignore Subsequent Working
SOI	Seen or implied
SR	Special Ruling (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)

Penalties

MR –1	A penalty of MR –1 is deducted from A or B marks when the data of a question or part question are genuinely misread and the object and difficulty of the question remain unaltered. In this case all A and B marks then become “follow through” marks. MR is not applied when the candidate misreads his own figures – this is regarded as an error in accuracy. An MR –2 penalty may be applied in particular cases if agreed at the coordination meeting.
PA –1	This is deducted from A or B marks in the case of premature approximation. The PA –1 penalty is usually discussed at the meeting.

Question	Answer	Marks	Guidance
1	$\frac{dx}{dt} = e^t - 1$ and $\frac{dy}{dt} = 2e^{\frac{1}{2}t}$	**B1**	
	$\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2 = \left(e^t + 1\right)^2$	**M1 A1**	M1 for using $\left(\frac{ds}{dt}\right)^2 = \left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2$
	Arc length is $\int_0^3 e^t + 1\,dt = \left[e^t + t\right]_0^3$	**M1**	M1 for good attempt at correct integral
	$= 2 + e^3$ (or 22.1)	**A1**	
		5	

Question	Answer	Marks	Guidance
2	We have that $f(1) = 9$ is divisible by 9.	**B1**	Checks base case.
	Assume that $f(k)$ is divisible by 9.	**B1**	Makes general statement.
	$f(k+1) + f(k) = 2^{3k+3} + 8^k + 2^{3k} + 8^{k-1} =$	**M1**	**Uses** expansion of $f(k+1)$.
	$2^3.2^{3k} + 8.8^{k-1} + 2^{3k} + 8^{k-1}$ OE	**A1**	Correct split of powers
	$= 9\left(2^{3k} + 8^{k-1}\right)$ OE so $f(k+1)$ is divisible by 9.	**A1**	Alt method: $f(k+1) = 2^{3k+3} + 8^k$
	So if f(k) is divisible by 9, so is $f(k+1)$, (and $f(1)$ is divisible by 9), $f(n)$ is divisible by 9 for every integer $n \geqslant 1$	**A1**	$= 2^3.2^{3k} + 8.8^{k-1}$ M1 $= 8f(k)$ A1
		6	

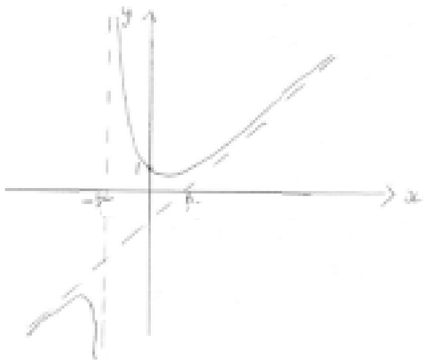
Question	Answer	Marks	Guidance
3(i)		**B1**	Correct position including label of 1 on initial line, and symmetric about initial
		B1	Single correct loop.
		2	
3(ii)	$\frac{1}{2}\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} cos^2 2\theta d\theta$	**M1**	Correct integral
	$= \frac{1}{4}\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \cos 4\theta + 1 d\theta =$	**M1**	Correct use of double angle formula
	$\frac{1}{4}\left[\frac{1}{4}\sin 4\theta + \theta\right]_{-\frac{\pi}{4}}^{\frac{\pi}{4}} = \frac{\pi}{8} = 0.393$	**A1**	
		3	

Question	Answer	Marks	Guidance
3(iii)	$r=\cos^2\theta-\sin^2\theta$ OE	**B1**	Uses trig identity
	Thus $\left(x^2+y^2\right)^{\frac{3}{2}}=x^2-y^2$	**M1 A1**	Uses $x=r\cos\theta$ or $y=r\sin\theta\, or\, both$, AEF
		3	

Question	Answer	Marks	Guidance
4(i)	$\alpha\beta\gamma=a^3=216\Rightarrow a=6$	**M1 A1**	Uses product of roots
	$a+ar+ar^{-1}=21$ $6\left(1+r+r^{-1}\right)=21$	**M1**	Uses sum of roots
	$2r^2-5r+2=0\Rightarrow r=2$ or $r=0.5$	**M1 A1**	Substitutes for a and solves quadratic
	Roots are 6, 12, 3	**A1**	
		6	
4(ii)	$k=\alpha\beta+\alpha\gamma+\beta\gamma=6(12)+6(3)+12(3)=126$	**M1 A1**	Or finds coefficient of x in $(x-3)(x-6)(x-12)$. Or substitutes root into equation
		2	

Question	Answer	Marks	Guidance
5(i)	$S_{2n} = 1^2 - 2^2 + 3^2 - 4^2 \ldots\ldots$	**M1**	Uses correct difference.
	so $S_{2n} = \sum_{r=1}^{2n} r^2 - 2\sum_{r=1}^{n} (2r)^2 = \sum_{r=1}^{2n} r^2 - 8\sum_{r=1}^{n} r^2$	**A1**	Alt method: Use $\sum_1^n (2r-1)^2 - \sum_1^n (2r)^2$ =A1
	Thus $S_{2n} = \frac{1}{6}(2n)(2n+1)(4n+1) - \frac{8}{6}n(n+1)(2n+1)$	**M1**	$\sum_1^n 4r^2 - 4\sum_1^n (r) + n - 4\sum_1^n (r)^2$ M1
	Factorising, $S_{2n} = \frac{1}{3}n(2n+1)(4n+1-4n-4) = -n(2n+1)$	**A1**	$= -n(2n+1)$ A1 AG
		4	
5(ii)	$\lim_{n\to\infty} \frac{S_{2n}}{n^2} = -2$	**B1**	
	$S_{2n+1} = S_{2n} + (-1)^{2n}(2n+1)^2$	**M1**	
	So, $S_{2n+1} = -n(2n+1) + (2n+1)^2 = (2n+1)(n+1)$	**M1**	Uses the result given in **(i)** or using $\lim_{n\to\infty} \frac{S_{2n}}{n^2}$ and correct sign
	Thus $\lim_{n\to\infty} \frac{S_{2n+1}}{n^2} = 2$	**A1**	Alt: Find limit from previous line directly
		4	

Question	Answer	Marks	Guidance
6(i)	Vertical asymptote is $x = -b$.	B1	
	$x^2 + b = (x+b)(x-b) + b^2 + b$ or $x+b\overline{)x^2+0x+b}$ with quotient $x-b$	M1	By inspection or long division.
	Thus the oblique asymptote is $y = x - b$	A1	
		3	
6(ii)	If $y = 0$ then $x^2 + b = 0$ which has no real root.	B1	Must refer to b > 0 OE
		1	
6(iii)	$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{2x(x+b) - (x^2+b)}{(x+b)^2} = 0 \Rightarrow x^2 + 2bx - b = 0$ Or differentiating $y = x - b + \frac{b^2+b}{x+b}$ and setting $\frac{\mathrm{d}y}{\mathrm{d}x} = 0$ gives $1 - \frac{b^2+b}{(x+b)^2} = 0$.	M1	Find $\frac{\mathrm{d}y}{\mathrm{d}x}$ and set = 0
	$b^2 + b > 0$ Therefore there are two stationary points on C	A1	Use discriminant or $(x + b)^2$ to show two stationary points
		2	

Question	Answer	Marks	Guidance
6(iv)		**B1**	Intersection (0,1) given and asymptotes drawn
		B1 B1	Each branch correct Penalise at most one mark for poor forms at infinity
		3	

Question	Answer	Marks	Guidance
7	The auxiliary equation is $\left(n+\frac{1}{7}\right)^2=0 \Rightarrow n=-\frac{1}{7}$.	**M1**	
	CF: $y=(A+Bx)e^{-\frac{1}{7}x}$.	**A1**	
	PI: $y=px+q$, $y'=p$ and $y''=0$.	**M1**	Correct form of PI and derivatives
	$14p+px+q=49x+735 \Rightarrow p=49,\ q=49$.	**M1 A1**	Substitutes in equation correctly
	Thus $y=(A+Bx)e^{-\frac{1}{7}x}+49(x+1)$.	**A1 FT**	FT only on correct form of CF AEF
	$y=0$ when $x=0$ gives $A+49=0 \Rightarrow A=-49$.	**B1**	
	$y'=-\frac{1}{7}(A+Bx)e^{-\frac{1}{7}x}+Be^{-\frac{1}{7}x}+49$.	**M1**	Differentiating their y
	$y'=0$ when $x=0$ gives $-\frac{1}{7}A+B+49=0 \Rightarrow B=-56$	**A1**	AEF
	Thus $y=-(49+56x)e^{-\frac{1}{7}x}+49(x+1)$.	**A1**	
		10	

Question	Answer	Marks	Guidance
8(i)	Reduces to row echelon form $\begin{pmatrix}1&2&\alpha&-1\\2&6&-3&-3\\3&10&-6&-5\end{pmatrix}\sim\begin{pmatrix}1&2&\alpha&-1\\0&2&-3-2\alpha&-1\\0&4&-6-3\alpha&-2\end{pmatrix}$	**M1**	Good attempt at REF
	$\sim\begin{pmatrix}1&2&\alpha&-1\\0&2&-3-2\alpha&-1\\0&0&\alpha&0\end{pmatrix}$	**A1**	
	Solves system of equations $x+2y+\alpha z-t=0$	**M1**	Forms system of equations from row echelon matrix
	$2y-(3+2\alpha)z-t=0$ $\alpha z=0$ Since $\alpha\neq 0$, $z=0$ and	**A1**	Three correct equations
	$(K_1=)\left\{\begin{pmatrix}0\\1\\0\\2\end{pmatrix}\right\}$	**A1**	AEF
		5	

Question	Answer	Marks	Guidance
8(ii)	If $\alpha = 0$ then $x + 2y - t = 0$ $2y - 3z - t = 0$	**M1**	Forms system of 2 equations
	$\Rightarrow K_2 = any\, 2\, of \begin{pmatrix} -6 \\ 3 \\ 2 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 1 \\ 0 \\ 2 \end{pmatrix} or \begin{pmatrix} 3 \\ 0 \\ -1 \\ 3 \end{pmatrix}$	**A1 A1**	AE
		3	
8(iii)	A basis vector for K_1 forms part of a basis for K_2 or a linear combination of basis vectors from K_2 form basis for K_1.	**M1**	Credit should be given for a correct conclusion using their bases.
	Therefore K_1 is a subspace of K_2.	**A1 FT**	
		2	

Question	Answer	Marks	Guidance
9(i)	$\frac{\mathrm{d}u}{\mathrm{d}x} = \sec^2 x$ so integral becomes $\int u^2\, du$	**M1**	Uses substitution correctly
	$= \frac{1}{3}\tan^3 x + C$	**A1**	
		2	
9(ii)	$I_n = \int_0^{\frac{\pi}{4}} \sec^{n-2} x \sec^2 x \tan^2 x$	**B1**	Separates into correct structure
	$= \frac{1}{3}\left[\sec^{n-2} x \tan^3 x\right]_0^{\frac{\pi}{4}} - \frac{n-2}{3}\int_0^{\frac{\pi}{4}} \sec^{n-2} x \tan^4 x dx$	**M1**	Uses integration by parts correctly
	$= \frac{2^{\frac{n-2}{2}}}{3} - \frac{n-2}{3}\int_0^{\frac{\pi}{4}} \sec^{n-2} x \tan^2 x\left(\sec^2 x - 1\right) dx$	**M1**	Uses $\tan^2 x = \sec^2 x - 1$
	$= \frac{2^{\frac{n-2}{2}}}{3} - \frac{n-2}{3} I_n + \frac{n-2}{3} I_{n-2}$	**A1**	
	Thus $(3 + n - 2) I_n = \left(\sqrt{2}\right)^{n-2} + (n-2) I_{n-2}$.	**A1**	AG
		5	

Question	Answer	Marks	Guidance
9(iii)	$I_2 = \left[\dfrac{\tan^3 x}{3}\right]_0^{\frac{\pi}{4}} = \dfrac{1}{3}$	**B1 FT**	Attempts to find I_2
	$5I_4 = 2 + 2I_2$	**M1**	Uses reduction formula
	Mean value $= \dfrac{4}{\pi} I_4 = \dfrac{32}{15\pi}$	**A1**	
		3	

Question	Answer	Marks	Guidance
10(i)(a)	Point on l_1 is $\begin{pmatrix} \lambda a \\ 9-\lambda \\ 2+\lambda \end{pmatrix}$ and on l_2 is $\begin{pmatrix} -6-\mu a \\ -5+2\mu \\ 10+4\mu \end{pmatrix}$	**B1**	
	Point of intersection: $\begin{pmatrix} \lambda a \\ 9-\lambda \\ 2+\lambda \end{pmatrix} = \begin{pmatrix} -6-\mu a \\ -5+2\mu \\ 10+4\mu \end{pmatrix}$	**M1**	Equates coordinates of points
	$\Rightarrow \mu = 1,\ \lambda = 12,\ a = -\dfrac{6}{13}$	**A1**	AG
		3	

Question	Answer	Marks	Guidance
10(i)(b)	Normal to the plane: $\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -\frac{6}{13} & -1 & 1 \\ \frac{6}{13} & 2 & 4 \end{vmatrix} = -6\mathbf{i} + \frac{30}{13}\mathbf{j} - \frac{6}{13}\mathbf{k} \sim -13\mathbf{i} + 5\mathbf{j} - \mathbf{k}$	**M1 A1**	Uses cross product to find normal to the plane AEF
	Using point on plane: e.g. $5(9) - 2 = 43$	**M1**	Substitutes a point
	Equation of plane: $-13x + 5y - z = 43$	**A1**	AEF
		4	

Question	Answer	Marks	Guidance
10(ii)	$\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ a & -1 & 1 \\ -a & 2 & 4 \end{vmatrix} = -6i - 5aj + ak$ $\left\| \begin{pmatrix} -6 \\ -5a \\ a \end{pmatrix} \right\| = \sqrt{36 + 26a^2}$	**B1**	Finds the magnitude of the cross product of the direction vectors of the lines
	$\begin{pmatrix} -6 \\ -5-9 \\ 10-2 \end{pmatrix} \cdot \begin{pmatrix} -6 \\ -5a \\ a \end{pmatrix} = 36 + 78a$	**M1 A1**	Takes dot product of the correct vectors
	$3\sqrt{30}\sqrt{36+26a^2} = \|36+78a\|$	**M1**	Puts distance equal to $3\sqrt{30}$
	$\Rightarrow 15\left(18+13a^2\right) = \left(6+13a\right)^2$ $\Rightarrow 26\left(a-3\right)^2 = 0 \qquad \Rightarrow \text{a} = 3$	**A1**	
		5	

Question	Answer	Marks	Guidance
11E(i)	If $z \neq -1$ then $\dfrac{z-1}{z+1} = \dfrac{z^{\frac{1}{2}} - z^{-\frac{1}{2}}}{z^{\frac{1}{2}} + z^{-\frac{1}{2}}} \left(\text{or} = \dfrac{2i\sin\theta}{2(\cos\theta+1)} \right)$	**M1 A1**	Using de Moivre's theorem or multiplying numerator and denominator by $\cos\theta + 1 - i\sin\theta$
	$= \dfrac{2i\sin\frac{1}{2}\theta}{2\cos\frac{1}{2}\theta} = i\tan\frac{1}{2}\theta$	**A1**	
		3	
11E(ii)	If $z = 1$ then $z^r - 1 = 0$ so the given sum is zero.	**B1**	
	If $z \neq 1$ then $z = e^{i\frac{2\pi}{3}}$ or $z = e^{-i\frac{2\pi}{3}}$, so	**B1**	Must consider all three roots of unity
	$\dfrac{z^3-1}{z^3+1} + \dfrac{z^2-1}{z^2+1} + \dfrac{z-1}{z+1} = 0 + \dfrac{e^{\pm i\frac{4\pi}{3}}-1}{e^{\pm i\frac{4\pi}{3}}+1} + \dfrac{e^{\pm i\frac{2\pi}{3}}-1}{e^{\pm i\frac{2\pi}{3}}+1}$	**M1 A1**	AEF two of three terms correct for M1
	Using **(ii)**, or by direct calculation, $\dfrac{e^{\pm i\frac{4\pi}{3}}-1}{e^{\pm i\frac{4\pi}{3}}+1} + \dfrac{e^{\pm i\frac{2\pi}{3}}-1}{e^{\pm i\frac{2\pi}{3}}+1} = i\tan\dfrac{1}{2}\left(\dfrac{4\pi}{3}\right) + i\tan\dfrac{1}{2}\left(\dfrac{2\pi}{3}\right) = 0$	**A1**	AG
		5	

Question	Answer	Marks	Guidance
11E(iii)	$z=1,\ e^{\pm i\frac{2\pi}{3}}$, $(z^3-1)(z^2+1)(z+1)+(z^2-1)(z^3+1)(z+1)+(z-1)(z^3+1)(z^2+1)=0$	**B1**	Writes down the cube roots of unity. AEF, must be exact
	$\Rightarrow 3z^6+z^5+z^4-z^2-z-3=0$	**M1**	Expands
	$3z^6+z^5+z^4-z^2-z-3=(z+1)(z^3-1)(3z^2-2z+3)$	**M1 A1**	Factorises
	$\Rightarrow z=-1,\quad \dfrac{2\pm i\sqrt{32}}{6}=\dfrac{1}{3}\pm i\dfrac{2}{3}\sqrt{2}$	**M1 A1**	Finds other three roots. AEF, must be exact
		6	

Question	Answer	Marks	Guidance
11O(i)	e.g. $2\mathbf{e}$	**B1**	Allow any scalar multiple $\mu\mathbf{e}$ where $\mu\neq 0,1$
		1	
11O(ii)	Eigenvector: $\mathbf{e}$, Eigenvalue: λ^n	**B1 B1**	Allow any scalar multiple $\mu\mathbf{e}$ where $\mu\neq 0$ Note: $\mathrm{A}^n\,\mathbf{e}=\lambda^n\,\mathbf{e}$ SCB1
		2	

Question	Answer	Marks	Guidance
11O(iii)	Eigenvalues of (diagonal matrix) $\mathbf{A}$: $\lambda = 3,7,1$ (Or from characteristic equation: $(\lambda-3)(\lambda-7)(\lambda-1)=0$)	**B1**	
	$\lambda = 3$: $\mathbf{e}_1 = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 2 & 4 & 0 \\ 4 & 8 & -2 \end{vmatrix} = \begin{pmatrix} -8 \\ 4 \\ 0 \end{pmatrix} = t\begin{pmatrix} -2 \\ 1 \\ 0 \end{pmatrix}$	**M1 A1**	Uses vector product (or equations) to find corresponding eigenvectors
	$\lambda = 7$: $\mathbf{e}_2 = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -4 & 0 & 0 \\ 4 & 8 & -6 \end{vmatrix} = \begin{pmatrix} 0 \\ -24 \\ -32 \end{pmatrix} = t\begin{pmatrix} 0 \\ 3 \\ 4 \end{pmatrix}$	**A1**	
	$\lambda = 1$: $\mathbf{e}_3 = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 2 & 6 & 0 \\ 4 & 8 & 0 \end{vmatrix} = \begin{pmatrix} 0 \\ 0 \\ -8 \end{pmatrix} = t\begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$	**A1**	
	Thus $\mathbf{P} = \begin{pmatrix} -2 & 0 & 0 \\ 1 & 3 & 0 \\ 0 & 4 & 1 \end{pmatrix}$ and $\mathbf{D} = \begin{pmatrix} 3^n & 0 & 0 \\ 0 & 7^n & 0 \\ 0 & 0 & 1 \end{pmatrix}$.	**B1 FT B1 FT**	Or correctly matched permutations of columns. FT on non-zero and distinct eigenvalues and vectors
		7	
11O(iv)	$\sum_{n=1}^{N}\left(k^n A^n - k^{n+1}A^{n+1}\right) = kA - k^{N+1}A^{N+1}$	**M1 A1**	Method of differences
	$k^{N+1}A^{N+1} \to 0$ as $N \to \infty$ for	**M1**	
	$-\frac{1}{7} < k < \frac{1}{7}$.	**A1**	
		4	